

Next-Generation Reservoir Computing for Hidden Variable Recovery in Kuramoto Oscillator Networks

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Abstract—The recovery of hidden macroscopic variables from complex dynamical systems is a fundamental challenge in network science and nonlinear dynamics. In this work, we apply the Next-Generation Reservoir Computing (NG-RC) framework to the reconstruction of collective variables in adaptive Kuramoto oscillator networks. By combining delay embedding with polynomial nonlinearities, NG-RC provides an efficient and robust approach to capturing macroscopic dynamics from partial observations. The results demonstrate that NG-RC outperforms classical reservoir computing methods, offering a promising tool for analyzing emergent behavior in large-scale oscillator networks.

Index Terms—Reservoir computing, Next-Generation Reservoir Computing, Kuramoto model, oscillator networks, adaptive networks

I. INTRODUCTION

Understanding and predicting the collective dynamics of complex systems remains one of the central problems in modern science and engineering. Many real-world systems, ranging from neural and social networks to power grids and biological oscillators, can be modeled as networks of coupled dynamical units. Despite the simplicity of local interactions, such systems often exhibit rich emergent behavior at the macroscopic scale such as collective synchronous behaviour [1], chimera states [2], [3], coherence resonance [4], [5], etc. The ability to reconstruct hidden macroscopic variables from limited observations is essential for revealing, predicting, and ultimately controlling the collective behavior of complex dynamical systems [6], [7].

Among the established models of collective synchronization, the Kuramoto model plays a central role. It provides a minimal yet powerful framework for studying how macroscopic order arises from microscopic phase interactions [8]–[10]. However, in realistic scenarios, only partial measurements of the system are typically available, making the problem of reconstructing hidden variables both practically relevant and computationally challenging [11], [12]. This motivates the development of methods that are able to efficiently extract global system properties from incomplete and noisy data.

Reservoir Computing (RC) has emerged as a promising paradigm for modeling and forecasting the dynamics of nonlinear systems due to its simplicity and computational

efficiency. Nevertheless, conventional RC architectures often require large reservoirs and extensive parameter tuning to achieve adequate accuracy. Recently, Next-Generation Reservoir Computing (NG-RC) has been proposed as an alternative framework that eliminates the need for recurrent dynamics by combining delay embedding with polynomial feature expansion [13].

In this work, we apply NG-RC to the recovery of hidden macroscopic variables in adaptive Kuramoto oscillator networks.

II. METHODS

Consider a network of $N = 300$ Kuramoto phase oscillators with phases $\phi_i(t)$ and natural frequencies ω_i . The microscopic dynamics are governed by [14], [15]:

$$\frac{d\psi_i(t)}{dt} = \omega_i + \sum_{j \neq i} \mathcal{W}_{ij}(t) \sin(\psi_j(t) - \psi_i(t)), \quad (1)$$

where λ is the coupling strength and $w_{ij}(t)$ represents the time-dependent connection weights described as

$$\frac{d\mathcal{W}_{ij}(t)}{dt} = \rho_{ij}(t) - \left[\sum_{k \neq i} \rho_{ik}(t) \right] \mathcal{W}_{ij}(t), \quad (2)$$

where

$$\rho_{ij}(t) = \frac{1}{T} \left| \int_{t-T}^t e^{\sqrt{-1}(\psi_i(\tau) - \psi_j(\tau))} d\tau \right|, \quad (3)$$

is the phase correlation between each pair (i, j) of phase oscillators over a time of observation T .

The evolving network structure resulting from the adaptation process is shown in Fig. 1. Panel (A) illustrates the initial, non-adapted configuration, in which each node is connected to $K = 20$ nearest neighbors, forming a densely interconnected and chaotically organized topology without clear modularity. Panel (B) shows the adapted configuration, where the network reorganizes into distinct clusters: intra-cluster connections remain dense, while inter-cluster connections become sparse. This structural transformation reflects the emergence of modular organization, which supports more coherent collective dynamics.

To analyze the macroscopic behavior, we partition the oscillators into $M = 6$ non-overlapping groups S_j ($j = 1, \dots, M$)

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with $M_{\text{group}} = 50$ oscillators each. The macroscopic signal for each group is defined as:

$$X_j(t) = \frac{1}{M_{\text{group}}} \sum_{i \in S_j} \sin[\psi_i(t)] \quad (4)$$

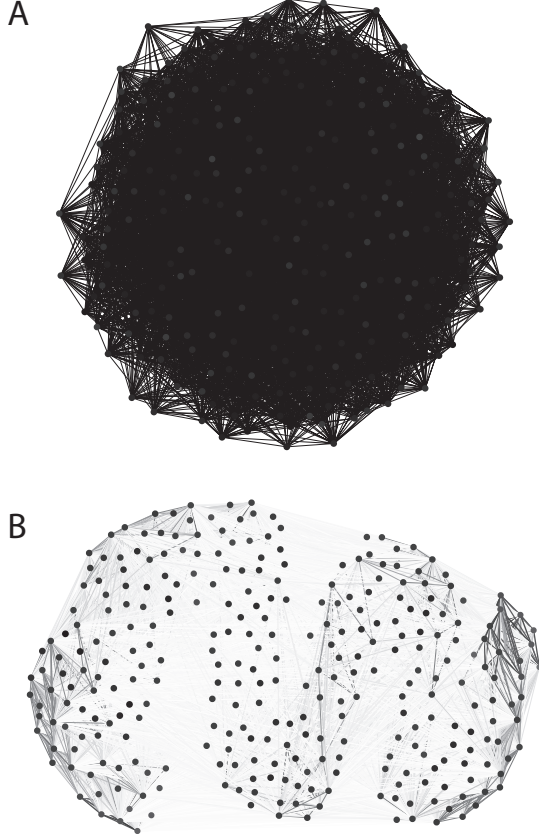


Fig. 1. Structure of the Kuramoto network before and after adaptation. (A) The initial, non-adapted network with relatively high connection density and a chaotic organization of interactions between nodes. (B) The adapted network, characterized by the division of nodes into clusters with dense intra-cluster connections and sparse inter-cluster connections.

In our reconstruction task, one randomly selected macroscopic signal serves as the target $X^{\text{target}}(t)$, while the remaining $M - 1$ signals constitute the observed inputs $\mathbf{X}^{\text{in}}(t) = [X_1(t), \dots, X_{j-1}(t), X_{j+1}(t), \dots, X_{M-1}(t)]^T$.

For the reconstruction task, we employed the Next-Generation Reservoir Computing (NG-RC) framework as proposed in [13]. We first conducted a systematic optimization of the model parameters to identify configurations that ensure stable and accurate performance. Based on this procedure, the following parameter set was selected: delay time $\Delta t = 100$, number of delays $N_{\text{delay}} = 21$, polynomial order $p = 1$, and ridge regularization coefficient 10^{-6} .

III. RESULTS

The performance of the proposed NG-RC approach was assessed in terms of its ability to reconstruct macroscopic dynamics of the adaptive Kuramoto network. Figure 2 summarizes the obtained results. Panel (A) displays the original

signal (orange) together with the reconstructed signal (blue), demonstrating their close correspondence over the entire time interval. Panel (B) shows the normalized absolute difference, which remains consistently low, thereby reflecting the accuracy of the reconstruction.

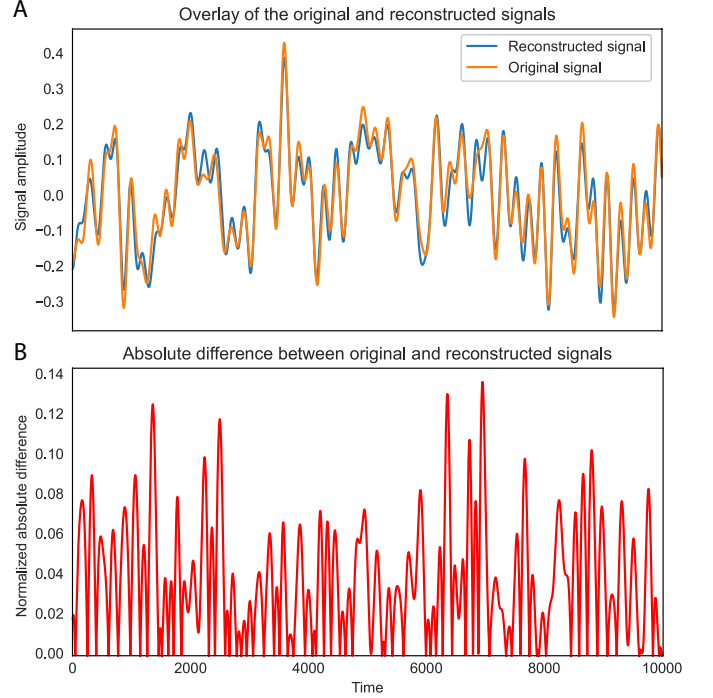


Fig. 2. Comparison of the original and reconstructed signals. The upper panel shows the overlay of the original signal (orange) and the reconstructed signal (blue). The lower panel illustrates the normalized absolute difference between the two signals, providing a measure of reconstruction accuracy over time.

To provide a quantitative evaluation, we computed the normalized root mean squared error (NRMSE) between the reconstructed and original signals. In the reported experiment, the NRMSE reached a value of 0.078, indicating high reconstruction accuracy. This performance represents a substantial improvement compared to our earlier studies and clearly outperforms the classical reservoir computing (RC) scheme. These results demonstrate that the NG-RC framework enables more accurate and robust recovery of macroscopic dynamics in oscillator networks [6].

IV. CONCLUSION

The results of this study demonstrate that the NG-RC framework provides an effective approach for recovering hidden macroscopic variables in adaptive Kuramoto networks. By combining polynomial nonlinearities with delay embedding, the method is capable of capturing essential phase coupling mechanisms while preserving temporal correlations that underlie oscillator synchronization. The use of a linear readout further ensures computational efficiency without compromising representational power.

Importantly, the achieved reconstruction accuracy highlights the ability of NG-RC to extract collective dynamics from

partial observations, despite the fact that macroscopic signals are highly nonlinear transformations of microscopic activity. These findings indicate that NG-RC can serve as a powerful tool for bridging microscopic and macroscopic scales in complex dynamical systems, offering both theoretical insights and practical utility for the analysis of oscillator networks and beyond.

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